

# Consistent histories and relativistic invariance in the modal interpretation of quantum mechanics

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## Abstract

Modal interpretations of quantum mechanics assign definite properties to physical systems and specify single-time joint probabilities of these properties. We show that a natural extension, applying to properties at several times, can be given if a decoherence condition is satisfied. This extension defines “histories” of modal properties. We suggest a modification of the modal interpretation, that offers prospects of a more general applicability of the histories concept. Finally, we sketch a proposal to apply the procedure for finding histories and a many-times probability distribution to the context of algebraic quantum field theory. We show that this leads to results that are relativistically invariant.

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# 1 Introduction

Modal interpretations of quantum mechanics [1, 2, 3, 4] interpret the mathematical formalism of quantum theory in terms of properties possessed by physical systems—in contradistinction to interpretations that take the formalism as an instrument to calculate macroscopic measurement outcomes and their probabilities. By the statement that a system possesses a property we mean that some quantum mechanical observable pertaining to the system takes on a definite value. However, it is impossible to give all observables definite values while preserving the quantum mechanical relations between them (as shown by the Kochen and Specker no-go theorem). Modal interpretations therefore specify a *subset* of all observables; only the observables in this subset are assigned definite values. It is characteristic of the modal approach that this is done in a state-dependent way: the quantum mechanical state of the system contains all information needed to determine the set of definite-valued observables. The precise prescription for finding this set makes use of the Schmidt bi-orthogonal decomposition of the composite state of a system plus its environment; or, more generally, of the spectral decomposition of the density operator describing a single system.

Most work on the modal interpretation up to now has concentrated on properties at a single time—with the exception of the pioneering studies of Bacciagaluppi and Dickson [6] and Vermaas [7]. No proposal for a joint probability distribution of physical properties at several times has been generally accepted, and it has been queried whether a natural joint distribution can exist at all [5]. Moreover, the probability distributions that have been proposed [6] suffer from the problem that they are not Lorentz-invariant; and it has been argued that it is a general feature of modal dynamical schemes that they single out a preferred frame of reference [8].

In this Letter we point out that in cases in which a decoherence condition is satisfied there is a natural and obvious candidate for a joint many-times probability distribution (in spite of the arguments to the contrary in [5]). We attempt to extend the applicability of this joint probability distribution by suggesting a modification of the modal interpretation scheme. Finally, we consider the application of the “histories” idea to quantum field theory (assuming decoherence again). Under certain conditions, the implementation of the idea here leads to a Lorentz-invariant joint probability distribution of properties associated with very small regions (approximating space-time points).

## 2 The modal scheme

Let  $\alpha$  be our system and let  $\beta$  represent its total environment (the rest of the universe). Let  $\alpha\&\beta$  be represented by  $|\psi^{\alpha\beta}\rangle \in \mathcal{H}^\alpha \otimes \mathcal{H}^\beta$ . The bi-orthonormal decomposition of  $|\psi^{\alpha\beta}\rangle$ ,

$$|\psi^{\alpha\beta}\rangle = \sum_i c_i |\psi_i^\alpha\rangle |\psi_i^\beta\rangle \quad , \quad (2.1)$$

with  $\langle \psi_i^\alpha | \psi_j^\alpha \rangle = \langle \psi_i^\beta | \psi_j^\beta \rangle = \delta_{ij}$ , generates a set of projectors operating on  $\mathcal{H}^\alpha$ :  $\{|\psi_i^\alpha\rangle\langle\psi_i^\alpha|\}_i$ . If there is no degeneracy among the numbers  $\{|c_i|^2\}$  this is a uniquely determined set of

one-dimensional projectors. If there is degeneracy, the projectors belonging to one value of  $\{|c_i|^2\}$  can be added to form a multi-dimensional projector; the thus generated new set of projectors, including multi-dimensional ones, is again uniquely determined. These projectors are the ones occurring in the spectral decomposition of the reduced density operator of  $\alpha$ .

The modal interpretation assigns definite values to the subset of all physical magnitudes that is generated by these projectors; i.e., the subset obtained by starting with these projectors, and then including their continuous functions, real linear combinations, and symmetric and antisymmetric products [16] (the thus defined real, closed in norm, linear subspace of all observables constitutes the set of “well-defined” or “applicable” physical magnitudes, in Bohrian parlance). *Which* value among the possible values of a definite magnitude is actually realised is not fixed by the interpretation. For each possible value a probability is specified: the probability that the magnitude represented by  $|\psi_i^\alpha\rangle\langle\psi_i^\alpha|$  has the value 1 is given by  $|c_i|^2$ . In the case of degeneracy it is stipulated that the magnitude represented by  $\sum_{i \in I_l} |\psi_i^\alpha\rangle\langle\psi_i^\alpha|$  has value 1 with probability  $\sum_{i \in I_l} |c_i|^2$  ( $I_l$  is the index-set containing indices  $j, k$  such that  $|c_j|^2 = |c_k|^2$ ).

The observation that the definite-valued projections occur in the spectral decomposition of  $\alpha$ 's density operator gives rise to a generalisation of the above scheme that is also applicable to the case in which the total system  $\alpha\&\beta$  is not represented by a pure state: find  $\alpha$ 's density operator by partial tracing from the total density operator, determine its spectral resolution and construct the set of definite-valued observables from the projection operators in this spectral resolution [9].

The above recipe for assigning properties is meant to apply to each physical system in a non-overlapping collection of systems that together make up the total universe [6, 10]. It is easy to write down a satisfactory *joint* probability distribution for the properties of such a collection (or a subset of it):

$$Prob(P_i^\alpha, P_j^\beta, \dots, P_k^\theta, \dots, P_l^\xi) = \langle \Psi | P_i^\alpha \cdot P_j^\beta \dots P_k^\theta \dots P_l^\xi | \Psi \rangle, \quad (2.2)$$

where the left-hand side represents the joint probability for the projectors occurring in the argument of taking the value 1, and where  $\Psi$  is the state of the total system consisting of  $\alpha, \beta, \theta$ , etc. [9]. It is important for the consistency of this probability ascription that the projection operators occurring in the formula all commute (which they do, since they operate in different non-overlapping Hilbert spaces).

However, a full-fledged interpretation of quantum mechanics in terms of properties of physical systems should not only specify the class of definite-valued observables at each instant, and their probability distribution, but should also make it clear what the probability is that a value present at one instant of time goes over into a given possible value at another time; or, equivalently, what the joint probability is of values of definite-valued observables at several times. It turns out that it is not easy to find a compelling and natural solution to this problem for the general case [6, 7]. But in the special, though physically important, case that a decoherence condition is satisfied there *is* such a natural joint probability for histories of properties, as we will now discuss.

### 3 Consistent histories of modal properties

There is a natural analogue of expression (2.2) for the case of Heisenberg projection operators pertaining to different instants of time:

$$\begin{aligned} \text{Prob}(P_i(t_1), P_j(t_2), \dots, P_l(t_n)) = \\ \langle \Psi | P_i(t_1) \cdot P_j(t_2) \cdot \dots \cdot P_l(t_n) \cdot P_l(t_n) \cdot \dots \cdot P_j(t_2) \cdot P_i(t_1) | \Psi \rangle. \end{aligned} \quad (3.1)$$

This expression is in accordance with the standard, “Copenhagen”, prescription for calculating the joint probability of outcomes of consecutive measurements. It agrees also with the joint distribution assigned to “consistent histories” in the consistent histories approach to the interpretation of quantum mechanics [11]. However, it should be noted that the projection operators in (3.1), pertaining to different times as they do, need not commute. As a result, (3.1) does not automatically yield a consistent probability distribution. For this reason it is an essential part of the consistent histories approach to impose the following decoherence condition, in order to guarantee that expression (3.1) is an ordinary Kolmogorov probability:

$$\begin{aligned} \langle \Psi | P_i(t_1) \cdot P_j(t_2) \cdot \dots \cdot P_l(t_n) \cdot P_{l'}(t_n) \cdot \dots \cdot P_{j'}(t_2) \cdot P_{i'}(t_1) | \Psi \rangle = 0 \\ i \neq i' \vee j \neq j' \vee \dots \vee l \neq l'. \end{aligned} \quad (3.2)$$

In the consistent histories approach all sequences of properties which satisfy the decoherence condition (3.2) are considered. A problem of this approach is that the decoherence condition leaves room for many, mutually incompatible, families of consistent properties, so that the set of definite-valued properties is not fully determined. By contrast, the modal interpretation provides an unequivocal rule to fix the definite-valued properties. It seems therefore worth-while to investigate whether the two approaches can be combined by using the above probability distribution for modal histories. However, in Ref. [5] it is argued that the projection operators singled out as definite-valued by the modal interpretation will not satisfy the decoherence condition (except in the very special and physically unrealistic circumstance in which the system’s properties evolve deterministically). If valid, this argument would make the probabilistic resources of the consistent histories approach unavailable to the modal interpretation.

The essential ingredient of the argument is the following (see the text accompanying Eq. (2.6) in Ref. [5]). Consider the decoherence condition pertaining to two instants,  $t_1$  and  $t_2$ , and let  $P_l(t_2) = |\psi_l(t_2)\rangle\langle\psi_l(t_2)|$ . We then have  $\langle \Psi | P_i(t_1) | \psi_l(t_2) \rangle \langle \psi_l(t_2) | P_{i'}(t_1) | \Psi \rangle = 0$ , if  $i \neq i'$ , for all values of  $l$ . That means, the argument goes, that either  $\langle \Psi | P_i(t_1) | \psi_l(t_2) \rangle = 0$  or  $\langle \psi_l(t_2) | P_{i'}(t_1) | \Psi \rangle = 0$ . From this it follows that  $|\psi_l(t_2)\rangle$  is orthogonal to all but one of the states  $|P_i(t_1)\Psi\rangle$ . That would imply a deterministic evolution of properties, and if the projectors  $P_i(t_1)$  are one-dimensional the properties at time  $t_2$  would even have to be the same as those at time  $t_1$ .

The questionable premise in this argument is the presupposition that  $\langle \Psi | P_i(t_1) | \psi_l(t_2) \rangle$  and  $\langle \psi_l(t_2) | P_{i'}(t_1) | \Psi \rangle$  are numbers, instead of a bra and a ket, respectively. Making the assumption that these expressions represent numbers is equivalent to assuming that  $|\Psi\rangle$

is an element of the Hilbert space of the system under consideration. This assumption does not fit in with the modal approach: the modal property ascription, as explained above, uses for  $|\Psi\rangle$  the state of the combination “system and rest of the universe”. As we shall show in a moment, the fact that the presence of an environment has always to be taken into account not only invalidates the above argument, but also makes it natural and easy to incorporate the idea of decoherence in the modal scheme so that condition (3.2) is satisfied. As a consequence, Eq. (3.1) yields a consistent joint multi-times probability distribution for modal properties in the case in which this decoherence condition is fulfilled.

The notion of decoherence to be used is the following. It is a general feature of the modal interpretation that if a system acquires a certain property, this happens by virtue of its interaction with the environment, as expressed in Eq. (2.1). As can be seen from this equation, in this process the system’s property becomes correlated with a property of the environment. Decoherence is now defined to imply the irreversibility of this process of correlation formation: the rest of the universe retains a trace of the system’s property, also at later times when the properties of the system itself may have changed. In other words, the rest of the universe acts as a memory of the properties the system has had; and decoherence guarantees that this memory remains intact. For the state  $|\Psi\rangle$  this means that in the Schrödinger picture it can be written in the following form:

$$|\Psi(t_n)\rangle = \sum_{i,j,\dots,l} c_{i,j,\dots,l} |\psi_{i,j,\dots,l}\rangle |\Phi_{i,j,\dots,l}\rangle \quad , \quad (3.3)$$

where  $|\psi_{i,j,\dots,l}\rangle$  is defined in the Hilbert space of the system,  $|\Phi_{i,j,\dots,l}\rangle$  in the Hilbert space pertaining to the rest of the universe, and where  $\langle\Phi_{i,j,\dots,l}|\Phi_{i',j',\dots,l'}\rangle = \delta_{ii'jj'\dots ll'}$ . In (3.3)  $l$  refers to the properties  $P_l(t_n)$ ,  $j$  to the properties  $P_j(t_2)$ ,  $i$  to the properties  $P_i(t_1)$ , and so on.

The physical picture that motivates a  $|\Psi\rangle$  of this form is that the final state results from consecutive measurement-like interactions, each of which is responsible for generating new properties. Suppose that in the first interaction with the environment the properties  $|\alpha_i\rangle\langle\alpha_i|$  become definite: then the state obtains the form  $\sum_i c_i |\alpha_i\rangle |E_i\rangle$ , with  $|E_i\rangle$  mutually orthogonal states of the environment. In a subsequent interaction, in which the properties  $|\beta_j\rangle\langle\beta_j|$  become definite, and in which the environment “remembers” the presence of the  $|\alpha_i\rangle$ , the state is transformed into  $\sum_{i,j} c_i \langle\beta_j|\alpha_i\rangle |\beta_j\rangle |E_{i,j}\rangle$ , with mutually orthogonal environment states  $|E_{i,j}\rangle$ . Continuation of this series of interactions eventually leads to Eq. (3.3), with in this case  $|\psi_{i,j,\dots,l}\rangle = |\psi_l\rangle$  (see below for a generalisation).

If this picture of consecutive measurement-like interactions applies, it follows that in the Heisenberg picture we have  $P_l(t_n) \dots P_j(t_2) P_i(t_1) |\Psi\rangle = c_{i,j,\dots,l} |\psi_{i,j,\dots,l}\rangle |\Phi_{i,j,\dots,l}\rangle$ . Substituting this in the expression at the left-hand side of Eq. (3.2), and making use of the orthogonality properties of the states  $|\Phi_{i,j,\dots,l}\rangle$ , we find immediately that the consistent histories decoherence condition (3.2) is satisfied. As a result, expression (3.1) yields a classical Kolmogorov probability distribution of the modal properties at several times.

## 4 A possible modification of the modal scheme

The just-discussed decoherence scenario is based on the assumption that in consecutive interactions the initial state of the system is not relevant for the properties that are generated: we wrote that  $|\alpha_i\rangle|E_i\rangle$  is transformed into  $\sum_j \langle\beta_j|\alpha_i\rangle|\beta_j\rangle|E_{i,j}\rangle$ , with the same set  $\{|\beta_j\rangle\}$  for all values of  $i$ . In some physically important cases this is not unrealistic. The prime example is the case in which the object is subjected to consecutive measurements by devices designed to measure certain observables, regardless of the state of the incoming system. A similar situation can occur in cases in which no instruments constructed by humans are present. For instance, interactions by means of a potential that depends only on the object observable  $A$ , with an environment with very many degrees of freedom, will tend to destroy the coherence between object states that correspond to different  $A$ -values and can thus be regarded as (approximate)  $A$ -measurements, regardless of the object's initial state. Consecutive exposures to such environments can be described by (3.3).

But in general we will have to consider the situation in which no such measurement-like interactions occur, and in which the type of the later interaction may depend on the result of an earlier interaction. In this general case we can represent what happens in an interaction by writing:

$$\sum_i c_i |\alpha_i\rangle |E_i\rangle \longrightarrow \sum_{i,j} c_{i,j} \langle\beta_{i,j}|\alpha_i\rangle |\beta_{i,j}\rangle |E_{i,j}\rangle, \quad (4.1)$$

with  $\langle\beta_{i,j}|\beta_{i,j'}\rangle = \delta_{jj'}$ . In this formula a biorthogonal decomposition has been written down for each value of  $i$  separately; in other words, we are looking at the various mutually orthogonal “branches”, that result from an interaction, separately and look how each of them branches itself in a subsequent interaction. A series of consecutive interactions described in this way again leads to a total final state of the form (3.3). However, the biorthogonal decomposition applied to the total right-hand side of (4.1) would now in general not yield the projectors  $|\beta_{i,j}\rangle\langle\beta_{i,j}|$  as definite-valued observables. Therefore,  $P_j(t_2).P_i(t_1)|\Psi\rangle$ , with  $P(t_{1,2})$  the projectors that *are* singled out by the biorthogonal decomposition as definite-valued at  $t_1$  and  $t_2$ , respectively, will not be equal to  $c_{i,j} \langle\beta_{i,j}|\alpha_i\rangle |\beta_{i,j}\rangle |E_{i,j}\rangle$  (the projectors  $P_j(t_2)$  will generally not commute with the projectors  $|\beta_{i,j}\rangle\langle\beta_{i,j}|$ ). This has the consequence that there is no reason to expect that the decoherence condition (3.2) will be satisfied.

This suggests a modification of the modal scheme, designed to guarantee that Eq. (3.1) remains valid as the joint probability of possessed properties, even in the more general situation just described. The idea is to change the property assignment in such a way that the right-hand side of Eq. (4.1) comes to represent a situation in which the projectors  $|\beta_{i,j}\rangle\langle\beta_{i,j}|$  *are* definite-valued. Such a property ascription would be in accordance with the intuition that the different branches are irrelevant to each other as long as no re-interference occurs. As in the many-worlds interpretation, in which we do not need to consider what happens in other worlds in order to describe what happens in our world, it is proposed here that the ascription of properties, as a result of interactions, should be done *per branch*.

To this end, we may posit as an interpretational rule that in a composite state of the form

$$\sum_{i,j} c_{i,j} \langle \beta_{i,j} | \alpha_i \rangle | \beta_{i,j} \rangle | E_{i,j} \rangle, \quad (4.2)$$

with  $\langle E_{i,j} | E_{i',j'} \rangle = \delta_{ii'jj'}$ , and  $\langle \beta_{i,j} | \beta_{i',j'} \rangle = \delta_{jj'}$ , resulting from an interaction of the form  $\sum_i c_i |\alpha_i\rangle |E_i\rangle \longrightarrow \sum_{i,j} c_{i,j} \langle \beta_{i,j} | \alpha_i \rangle | \beta_{i,j} \rangle | E_{i,j} \rangle$ , the projectors  $|E_{i,j}\rangle \langle E_{i,j}|$  represent properties of the environment, and that the system has properties  $|\beta_{i,j}\rangle \langle \beta_{i,j}|$ , in one-to-one correlation to these environmental properties. In terms of the biorthogonal decomposition, the new proposal says that the system's properties are determined by the separate decomposition that can be written down for each value of  $i$ : the terms of a biorthogonal decomposition that do not recombine (interfere) in subsequent interactions are treated as individual branches, isolated from the other terms. The various branches are assigned definite-valued observables through their own individual biorthogonal decomposition.

It should be noted that the state  $|\Psi\rangle$  of the total system, together with the total Hamiltonian, uniquely determine the properties ascribed in this scheme. This is because the Hamiltonian governs the evolution, so that the initial state—before interactions started—is fixed by  $|\Psi\rangle$  and the Hamiltonian. Further, the branching that occurs in the subsequent interactions is fully determined by writing down the biorthogonal decomposition (per branch) after each interaction.

According to this way of interpreting the quantum mechanical state, the properties  $P_i(t_n)$  that are assigned satisfy Eq. (3.2) and make Eq. (3.1) a consistent probability distribution (if there is decoherence, that is, as long as there is no interference of the several branches). This brings the modal scheme closer to the consistent histories interpretation, in which exactly those histories are considered for which Eq. (3.1) yields a consistent probability distribution. There would still be a distinction, though: the requirement that Eq. (3.1) be a consistent probability is by itself not enough to determine what properties are definite at the various instants—in other words, what the family of consistent histories is. There can be many mutually inconsistent but partially overlapping families of consistent histories [12]). This constitutes a problem in the consistent histories approach, because the consistency condition is the only constraint one has in that approach. By contrast, in our suggested modified modal scheme we retain the distinctive modal feature that the definite-valued observables are uniquely defined by a fixed rule. As a consequence, there is only one family of consistent histories to consider.

We do not further discuss and elaborate this suggested modification of the modal scheme here, but instead turn to the question of whether the notion of histories of definite properties, with well-defined probabilities, can also be applied outside of the context of non-relativistic quantum theory, to relativistic quantum field theory. It seems clear that any interpretation of quantum theory should at least have the prospect of being so applicable in order to be taken seriously.

## 5 Application to relativistic quantum field theory

The traditional interpretational problems of quantum mechanics exist no less in quantum field theory; and as we will see shortly there are also additional problems. Because quantum field theory occupies a central place in the present physical description of the world, any interpretation of quantum theory should at least offer prospects of leading to sensible results if applied to this new context. We will therefore now briefly outline a method to implement the modal ideas of the foregoing to quantum field theory.

As before, the central issue is that it is not obvious that the theory is about objective physical states of affairs, even in circumstances in which no macroscopic measurements are being made. The fields in quantum field theory do not attach values of physical magnitudes to space-time points. Rather, they are fields of operators, with a standard interpretation in terms of macroscopic measurement results. In accordance with what we have said about the interpretation of the Schrödinger-Heisenberg theory we would like to give another meaning to the formalism, namely in terms of physical systems that possess certain properties. We will take as our framework the formalism of local algebraic quantum field theory as explained in [13], because of its generality. In this framework, a  $C^*$ -algebra of observables is associated to each open region of Minkowski space-time. What we would like to do is to provide an interpretation in which not only operators, but also *properties* are assigned to space-time regions. That is, we would like at least some of the observables to have definite values. This would lead to a picture in which it is possible to speak of objective *events* (if some physical magnitude takes on a definite value in a certain spacetime region, this constitutes the event that this magnitude has that value there and then).

If the open space-time regions could be regarded as physical systems, each one of them described in a factor space of a total Hilbert space (this total Hilbert space would then be the tensor product of the spaces belonging to non-overlapping subsystems), an immediate generalisation of the modal scheme would be possible. However, algebraic quantum field theory is much less amicable to the notion of a localised physical system than might be expected. The local algebras are of type III, and this implies that they cannot be represented as algebras of bounded observables on a Hilbert space (such algebras are of type I). We can therefore not take the open space-time regions and their algebras as fundamental, if we want an interpretation in terms of (more or less) localised systems whose properties would specify an event. Such an interpretation is highly desirable, however, at least in the limiting situation in which classical concepts become applicable: it should be possible, in this limiting situation, to speak of field values in small space-time regions.

One possible way out is to use the algebras of type I that “lie between two local algebras”. That such type-I algebras exist is assumed in the postulate of the “split property” ([13], Ch. V.5, [14]). We accept this postulate and focus on the algebras of type I lying between two concentric standard “diamond” regions with radii  $r$  and  $r + \epsilon$ , respectively, with  $r$  and  $\epsilon$  small numbers. Two of such type-I algebras, defined with respect to two double diamonds associated with regions that have space-like separation, are independent



in a strong sense: the total algebra generated by them can be represented by the tensor product of the two algebras, defined on the tensor product of two representative Hilbert spaces  $\mathcal{H}_A$  and  $\mathcal{H}_B$  of the two individual algebras separately:  $\mathcal{H}_{A\&B} = \mathcal{H}_A \otimes \mathcal{H}_B$  [14].

In this way we may justify the approximate validity of the notion of a small space-time region as a physical subsystem represented in a factor space of the total Hilbert space. Of course, there is arbitrariness here, for example in choosing the positions of the double diamonds in the manifold and in fixing the values of  $r$  and  $\epsilon$ ; it is not to be expected that a sensible algebra will result if  $r \rightarrow 0$ . This reflects the fact that the structure of quantum field theory, despite first appearances, by itself does not provide a natural arena for a space-time picture in which physical magnitudes attach to small space-time regions. We expect therefore that a space-time interpretation, like the one we are trying to construct, will not have a fundamental status but can only be employed in a classical limiting situation in which classical field and particle concepts become approximately applicable. In other words, we do not assume that actually field values *are* associated with space-time points, and that our description provides an approximation to that real state of affairs. Rather, we think that in the general case the ordinary space-time picture will not be possible and that the classical field and particle concepts can only be applied in a limiting situation; accordingly, in the general case  $x$  and  $t$  will be parameters that do not possess their usual space-time meaning. The approximate applicability of classical concepts in a limiting situation is perhaps connected to the presence of decoherence mechanisms that decouple certain  $x$ ,  $t$  regions from each other (see also below; we will need such an assumption of decoherence anyway, to arrive at consistent joint probabilities).

We will now adopt the modal ideas of section 3 to assign values to the observables in the just-defined type-I algebras, loosely associated with small space-time regions. We will therefore assume that there is decoherence in the sense discussed in section 3. This assumption is natural in the context of local quantum physics [15], because the physical systems in local quantum physics automatically have an infinite environment to interact with, so that decoherence and irreversibility can easily occur.

The core idea of the modal interpretation is to select a subset of definite-valued observables from the algebra of all observables by means of an objective, fixed, rule. This was implemented within the framework of non-relativistic quantum mechanics via the selection principle based on the biorthogonal decomposition of the total state or, more generally, the spectral resolution of the system's density operator. Here, we will analogously consider the projectors occurring in the spectral resolution of the density operator (defined in a factor space) representing the partial state defined on the type-I algebra of observables attached to the very small "split-regions" that approximate space-time points. At each instant, we will consider a collection of such regions that have space-like separation with respect to each other, and can be described by means of the tensor product of the individual factor spaces. The selected projectors provide us with a base set of definite-valued quantities. As before, the complete collection of definite-valued observables can be constructed from this base set by closing the set under the operations of taking continuous functions, real linear combinations, and symmetric and antisymmetric products [16]. The probability of projector  $P_l$  having the value 1 is  $\langle \Psi | P_l | \Psi \rangle$ . Subdividing (approximately)

the whole of Minkowski space-time into a collection of non-overlapping point-like regions, that have space-like separation on each simultaneity hyperplane, and applying the above prescription to the associated algebras, we achieve the picture aimed at: to each (approximate) space-time point belong definite values of some physical magnitudes, and this constitutes an event at the position and time in question.

In order to complete this picture we should specify the joint probability of events taking place at different space-time “points”. It is natural to consider, for this purpose, a generalisation of expression (3.1). The first problem encountered in generalizing this expression to the relativistic context is that we no longer have absolute time available to order the sequence  $P_i(t_1), P_j(t_2), \dots, P_l(t_n)$ . In Minkowski space-time we only have the partial ordering  $y < x$  (i.e.,  $y$  is in the causal past of  $x$ ) as an objective relation between space-time points. However, we can still impose a linear ordering on the space-time points in any region in space-time by considering equivalence classes of points which all have space-like separation with respect to each other—for instance, points whose centres are on the same simultaneity hyperplane. Of course, there are infinitely many ways of subdividing the region into such space-like collections of points. It will have to be shown that the joint probability distribution that we are going to construct is independent of the particular subdivision that is chosen.

Take one particular linear time ordering of the points in a closed region of Minkowski space-time, for instance one generated by a set of parallel simultaneity hyperplanes (i.e. hyperplanes that are all Minkowski-orthogonal to one given time-like worldline). Let the time parameter  $t$  label very thin slices of space-time (approximating hyperplanes) in which “points” of the kind introduced above, with mutual space-like separation, are located. We can now write down a joint probability distribution for the properties, in exactly the same form as in Eq. (3.1):

$$\begin{aligned} \text{Prob}(P_i^*(t_1), P_j^*(t_2), \dots, P_l^*(t_n)) = \\ \langle \Psi | P_i^*(t_1) \cdot P_j^*(t_2) \cdot \dots \cdot P_l^*(t_n) \cdot P_l^*(t_n) \cdot \dots \cdot P_j^*(t_2) \cdot P_i^*(t_1) | \Psi \rangle. \end{aligned} \quad (5.1)$$

In this formula the projector  $P_m^*(t_l)$  represents the properties of the space-time “points” on the “hyperplane” labeled by  $t_l$ . That is:

$$P_m^*(t_l) = \Pi_i P_{m_i}(x_i, t_l), \quad (5.2)$$

with  $\{x_i\}$  the central positions of the point-like regions considered on the “hyperplane”. The index  $m$  is symbolic for the set of indices  $\{m_i\}$ . As explained above, the modal proposal is that the projectors  $P_{m_i}(x_i, t_l)$  come from the spectral decomposition of the mixed state defined on the algebra belonging to the space-time “point”  $(x_i, t_l)$  (we do not pursue here the change in this if we consider the modification suggested in section 4). Because all the considered point-like regions on the “hyperplane”  $t_l$  are space-like separated from each other, the associated projectors commute (the principle of micro-causality). This important feature of local quantum physics guarantees that the product operator of Eq. (5.2) is again a projection operator, so that expression (5.1) can be treated in the same way as Eq. (3.1). In particular, we will need an additional condition to ensure that (5.1) yields a Kolmogorovian probability.

The decoherence condition that we are going to use is essentially the same as discussed in section 3. Suppose that at space-time “point”  $(x, t)$  the magnitude represented by the set of projector operators  $\{P_k\}$  is definite-valued;  $P_l$  has value 1, say. Intuitively speaking, the notion of decoherence that we invoke is that in the course of the further evolution there subsists a trace of  $P_l(x, t)$  in the future lightcone of  $(x, t)$ . That is, decoherence implies that on each space-like hyperplane intersecting the future lightcone of  $(x, t)$  there is at least one space-time point, within this lightcone, that has a property strictly correlated to  $P_l$ . One way of fulfilling this decoherence condition would be given by the existence of a time-like worldline (a propagating signal) going through  $(x, t)$  (or a bundle of such worldlines), each point of which is characterized by the value 1 of  $P_l(x, t)$  (apart from evolution). This makes explicit the idea that the effects of the irreversible interaction responsible for decoherence propagate within the future lightcone of  $(x, t)$ .

If this decoherence condition is fulfilled, we have that on the hyperplane  $t_2$  at least one of the space-time points, say  $(x', t_2)$ , has properties  $\{P'_k\}$  correlated to  $\{P_k\}$ :  $P'_k.P_{k'}|\Psi\rangle = 0$  if  $k \neq k'$ .

In that case we not only have that that  $\langle\Psi|P_l(x, t_1)P'_k(x', t_2)P'_{k'}(x', t_2)P_l(x, t_1)|\Psi\rangle = 0$  if  $k \neq k'$  (this follows simply from the orthogonality of  $P_k$  and  $P_{k'}$ , expressing the incompatibility of two different values of the same observable at one space-time point), but we also find  $\langle\Psi|P_l(x, t_1)P'_k(x', t_2)P'_{k'}(x', t_2)P_{l'}(x, t_1)|\Psi\rangle = 0$  if  $l \neq l'$ , both if  $k = k'$  and if  $k \neq k'$ . This expresses that possibilities characterized by different values of an observable remain incompatible in the course of time. Assuming decoherence for the properties of all space-time points, we find the analogue of (3.2):

$$\begin{aligned} \langle\Psi|P_i^*(t_1).P_j^*(t_2).....P_l^*(t_n).P_{l'}^*(t_n).....P_{j'}^*(t_2).P_{i'}^*(t_1)|\Psi\rangle = 0 \\ i \neq i' \vee j \neq j' \vee .... \vee l \neq l'. \end{aligned} \quad (5.3)$$

This makes (5.1) a consistent Kolmogorovian joint probability for the joint occurrence of the events represented by  $P_i^*(t_1), P_j^*(t_2), ....P_l^*(t_n)$ .

The projectors  $P_i^*(t_1), P_j^*(t_2), ....P_l^*(t_n)$  depend for their definition on the chosen set of hyperplanes, labeled by  $t$ . Therefore (5.1) is not manifestly Lorentz invariant. However, the projectors  $P^*(t)$  are products of projectors pertaining to the individual space-time points lying on the  $t$ -hyperplanes, so (5.1) can alternatively be written in terms of these latter projectors. The specification of the joint probability of the physical quantities at all considered “points” in a given space-time region requires (5.1) with all individual projectors appearing in it. Depending on the way in which the space-time region has been subdivided in space-like hyperplanes in the definition of  $P_i^*(t_1), P_j^*(t_2), ....P_l^*(t_n)$ , the individual projectors occur in different orders in this complete probability specification. However, there is a lot of conventionality in this ordering. All operators attached to point-like regions with space-like separation commute, so that their ordering can be arbitrarily changed. The only characteristic of the ordering that is invariant under all these allowed permutations is that if  $y < x$  (i.e.  $y$  is in the causal past of  $x$ ),  $P(y)$  should appear before  $P(x)$  in the expression for the joint probability. But this is exactly the characteristic that is common to all expressions that follow from writing out (5.1), starting from all different ways of ordering events with a time parameter  $t$ . All these

expressions can therefore be transformed into each other by permutations of projectors belonging to space-time “points” with space-like separation. The joint probability thus depends only on how the events in the space-time region are ordered with respect to the Lorentz-invariant relation  $<$ ; it is therefore Lorentz-invariant itself.

## 6 Conclusion

The main aim of this Letter has been to show that a decoherence condition, analogous to the one whose fulfilment is assumed in the consistent histories approach, can be satisfied in the modal interpretation of quantum mechanics. If the condition is satisfied, a simple and natural joint probability can be specified for the values of definite-valued observables at several times. This probability expression is the same as the one proposed in the consistent histories scheme, or in traditional quantum measurement theory. It is here combined with the characteristic feature of the modal interpretation, namely that the set of definite-valued observables at each instant is determined by an objective rule that uses only the form of the quantum mechanical state; this rule selects one family of consistent histories.

Because quantum field theory is central in present-day theoretical physics, we have proposed a way of applying the modal histories scheme to that theory. This proposal was motivated by the desire to obtain, at least in the classical limiting situation, a picture in which events occur, in small space-time regions approximating space-time points. In this way it becomes possible in principle to recover the classical picture according to which field values are attached to space-time points. We have argued that the assumption of irreversible decoherence, combined with the modal ideas, indeed offers prospects of obtaining a Lorent-invariant account in which field magnitudes take on definite values in point-like regions and in which a consistent joint probability for such values can be defined.

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